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Editors’ foreword

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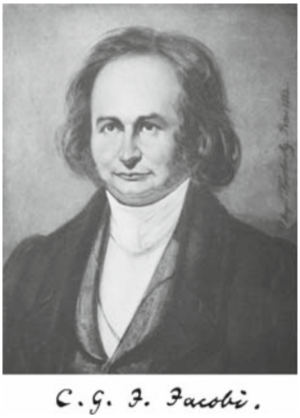

Portrait of Jacobi by
August Theodor Kaselowsky (Potsdam, 1810; Berlin, 1891),
painted in Roma in 1843.
Kaselowsky lived in Roma from 1839 to 1850.

This special issue is dedicated, on the first anniversary of his death, to the memory of Prof. E. V. Pankratiev.

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Computation of normal forms using a sequence of derivations and eliminations, change of orderings, resolvents, characterization of possible normal forms by the rank of jacobian matrices, a priori bounds on the order of a system, … the posthumous papers of Jacobi develop many themes quite familiar to contemporary research in differential computer algebra.

Algorithmic issues and efficiency considerations are explicitely present in this work, which is all the more of interest for computer theorists, as it contains the first polynomial algorithm solving the assignment problem. It was developed more than a century before it was rediscovered by Harold W. Kuhn under the name of "Hungarian method"!

This atypical historical special issue of AAECC presents the first English translations of two posthumous papers of Jacobi, completed with some unpublished material [II/13 b) 2203–2206; II/23 a), 2217–2220] discovered in Jacobi's Nachlaß (Archiv der Berlin-Brandenburgische Akademie der Wissenschaften). These two documents cover most of Jacobi's results related to the order and normal forms of ordinary differential systems.

They are completed with two contemporary papers, one giving a proof of Jacobi's bound for ordinary and partial differential systems, under Johnson's regularity hypothesis. The second is devoted to the computation of representations, without factorization, of radical differential ideals, as an intersection of radical ideals defined by characteristic sets. Besides the general guideline of normal forms and order computation, these two papers have in common with Jacobi's to have been somehow neglected.

The first contribution of Marina V. Kondratieva, Alexandr V. Mikhalev and Evgeniĭ V. Pankratiev to the study of Jacobi's bound appeared in Russian in 1982 [6], but it was only in recent years that this result and its many possible applications were noticed. In his obituary of Joseph Fels Ritt [9], Lorch wrote that he knew Russian at a time when it was not yet fashionable. It seems unfortunately that the fashion has gone. However, the contribution to this issue is a completely new paper, using improved mathematical tools that allow to generalize the bound to quasi-regular partial differential systems [7].

One cannot say that the paper of Boulier et al. was neglected and, to some extend, it has inspired many subsequent works during the last decade, but it remained unpublished after having failed to take place in some special issue of JSC. It seemed useful to ask the authors a last effort to insure the perenniality of their work. A foreword to that paper explains the changes made and provides updated references on the subject.

Jacques Calmet took in charge the editing process of the two translations and of the paper to which contributed F. Ollivier. He had the difficult task of finding competent referees for the translations, able to understand both Latin and mathematics. Two reports were obtained and it is a pleasure to thank Daniel J. Katz who did an outstanding job. F. Ollivier was the editor of the remaining contribution.

A little more history. It is not the place here to develop the long history of Jacobi's bound. An historical survey, containing an almost complete list of references should appear soon [10]. A "translation" of Jacobi's main results in the language of differential algebra, completed with proofs, is planed to appear in a next issue of AAECC. We will limit here to recall the most important steps in the transmission and oblivion of the bound and relative results. It seems that Jacobi worked on this subject between 1836 and 1845 but he published nothing in his lifetime. There is a single allusion to his *shortest reduction* method in [5]. After his death, his widow gave his manuscripts to his colleagues of the Berlin Academy, where they are still kept.[1] The publication process was long, perhaps due to the great disorder of the manuscripts or to the premature death of Sigismund Cohn, who managed to prepare a first transcription for publication [II/13 c)]. Borchardt achieved the publication of [3] in 1865 and certainly worked [II/25] on the manuscript of [4], which appeared in the volume [VD], edited by Clebsch in 1866.

Although known names of mathematics, such as Jordan, worked on the subject, nothing substantial was done before Ritt's paper [12], in which a first proof is given for arbitrary linear systems and for two variables. In 1955, Harold Kuhn rediscovered Jacobi's algorithm for solving the assigment problem. In 1970, Barbara Lando proved the *weak bound* for order one systems [8]. In 1960, Volevich rediscovered Jacobi's bound in the linear case [13]. In 1983, Richard Cohn proved that Jacobi's bound implies the dimension conjecture[2] and was the first to mention the existence of Jacobi's algorithm [1].

In 2001, Pryce rediscovered Jacobi's shortest reduction method and the *truncated determinant* criterion [11]. In 2004, Hrushovski gave a proof of Jacobi's bound for difference systems [2].

Jacques Calmet, François Ollivier
Karlsruhe, Palaiseau,
23rd January 2009.

1 Primary material

1.1 Manuscripts

[II/13 b)] *De ordine systematis aequationum differentialium canonici variisque formis quas inducere potest*. Manuscript by Jacobi. Folios: 2186–2196, 2200–2206. 35 p. The basis of Cohn's transcription [II/13 c)] and [3].

[II/13 c)] Sigismund Cohn, transcription of [II/13 b)] with corrections and notes by C. W. Borchardt, 39 p.

[II/23 a)] *Reduction simultaner Differentialgleichungen in ihre canonische Form und Multiplicator derselben.*, manuscript by Jacobi, pp. 2214–2237. Five different fragments: 2214–2216; 2217–2220 (Sects. 17–18); 2221–2225 (Sect. 17); 2226–2229; 2230–2232, 2235, 2237, 2236, 2238 (numbered from 1 to 13).

[II/23 b)] *De aequationum differentialium systemate non normali ad formam normalem revocando*, manuscript by Jacobi p. 2238, 2239–2241, 2242–2251. 25 p. Envelop by Borchardt. The basis of [4].

[II/25] *De aequationum differentialium systemate non normali ad formam normalem revocando*. Abstract and notes by Borchardt. 8 p.

[1] Except letters: some letters once in Berlin, such as letters from Legendre are now in the library of the Mittag-Leffler institute.

[2] Components defined by s differential equations are of codimension at most s.

1.2 Journals and complete works

[Crelle 27] [[Crelle 27]] *Journal für die reine und angewandte Mathematik*, **27**, 1844.
[Crelle 29] *Journal für die reine und angewandte Mathematik*, **29**, 1845.
[Crelle 64] *Journal für die reine und angewandte Mathematik*, **64**, Berlin, Georg Reimer, 1865.
[GW IV] *C.G.J. Jacobi's gesammelte Werke IV*, K. Weierstrass ed., Berlin, Georg Reimer, 1890.
[GW V] *C.G.J. Jacobi's gesammelte Werke V*, K. Weierstrass ed., Berlin, Georg Reimer, 1886.
[VD] *Vorlesungen über Dynamik von C. G. J. Jacobi nebstes fünf hinterlassenen Abhandlungen desselben*, A. Clesch ed., Berlin, Georg Reimer, 1866.

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3. Jacobi, C.G.J.: De investigando ordine systematis aequationum differentialum vulgarium cujuscunque, [Crelle 64], vol. 4, pp. 297–320, [GW V] pp. 193–216. English translation (this issue of AAECC)
4. Jacobi, C.G.J.: De aequationum differentialum systemate non normali ad formam normalem revocando, [VD] pp. 550–578, [GW V] pp. 485–513. English translation (this issue of AAECC)
5. Jacobi, C.G.J.: Theoria novi multiplicatoris systemati aequationum differentialium vulgarium applicandi, first published in [Crelle 27], vol. 3, pp. 199–268 (part I), [Crelle 29], vol. 3, pp. 213–279, vol. 4, pp. 333–376 (part II), reproduced in [GW IV], pp. 317–509
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